MECHANICAL PROPERTIES OF SOILS TAKING ACCOUNT OF THEIR VISCOPLASTIC PROPERTIES WITH SHORT-TERM DYNAMIC LOADS

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It is assumed that the compressibility of the soil with monaxial compression is described by a deformation law of the form

$$\frac{\partial\varepsilon}{\partial t} = G(\sigma_1 - f(\varepsilon)) + \begin{cases} \frac{1}{E(\varepsilon)}\frac{\partial\sigma_1}{\partial t}, & \frac{\partial\sigma_1}{\partial t} \geqslant 0, \\ \frac{1}{E_*(\sigma,\varepsilon)}\frac{\partial\sigma_1}{\partial t}, & \frac{\partial\sigma_1}{\partial t} < 0, \end{cases} \tag{1}$$

where G, E, E_*, f are some monotonically rising functions of its arguments; G > 0 with $\sigma_1 - f(\varepsilon) > 0$; $G \equiv 0$ with $\sigma_1 - f(\varepsilon) \leq 0$.

From (1) with $\sigma_1 > f(\varepsilon)$, $\partial\sigma_1/\partial t > 0$ and $\dot\varepsilon = \infty$, $\dot\varepsilon = 0$, we have, respectively, dynamic and static limiting diagrams of the compression with loading,

$$\sigma_1 = \int_0^\varepsilon E(\xi)\,d\xi \equiv \varphi(\varepsilon), \quad \dot\varepsilon = \infty; \tag{2}$$

$$\sigma_1 = f(\varepsilon), \ \dot\varepsilon = 0. \tag{3}$$

If $E_* = E_*(\varepsilon)$, then with $\partial\sigma_1/\partial t < 0$ and $\sigma_1 < f(\varepsilon)$, from (1) we obtain

$$\sigma_1 = \int_{\varepsilon_*}^{\varepsilon} E_*(\xi)\,d\xi + \sigma_{1*} \equiv \varphi_*(\varepsilon, \varepsilon_*). \tag{4}$$

Dependence (4) is a diagram of the compression with unloading, which, with $\sigma_1 < f(\varepsilon)$, does not depend on the rate of deformation. Here σ_{1*} and ε_* are the stress and the deformation attained in a particle at the moment determined by the condition $\sigma_1 = f(\varepsilon)$.

With $E_* = E = \text{const}$, (1) goes over into the Sokolovskii–Malvern model [3, 4]. With application to the propagation of explosion waves in soils, a model of this type was discussed in [5].

From (1)-(4) it follows that the mechanical characteristics of soils with monaxial compression, subject to experimental determination, are the functions $\varphi(\varepsilon)$, $\dot\varepsilon = \infty$; $f(\varepsilon)$, $\dot\varepsilon = 0$; $\varphi_*(\varepsilon, \varepsilon_*)$ and $G(\sigma_1 - f(\varepsilon))$.

In addition, with monaxial compression under conditions of plane deformation, with a simultaneous change in the two principal stresses σ_1 and σ_2, the function $F(\sigma) = k\sigma + b$ can be defined, characterizing the condition of plasticity [6-9]. In the case where the effect of the rate of deformation on the condition of plasticity is insignificant, from (1) we obtain the deformation law with volumetric compression [10],

$$\frac{\partial\varepsilon}{\partial t} = G_0(\sigma - f_0(\varepsilon)) + \begin{cases} \frac{1}{E^0(\varepsilon)}\frac{\partial\sigma}{\partial t}, & \frac{\partial\sigma}{\partial t} \geqslant 0, \\ \frac{1}{E^0_*(\sigma,\varepsilon)}\frac{\partial\sigma}{\partial t}, & \frac{\partial\sigma}{\partial t} < 0, \end{cases} \tag{5}$$

where $\sigma = (1/3)(\sigma_1 + 2\sigma_2) = \alpha\sigma_1 - b_1$, $\alpha = 3(3 + \sqrt{2}k)^{-1}$, $b_1 = \sqrt{2}b(3 + \sqrt{2}k)^{-1}$; $E^0(\varepsilon) = \alpha E(\varepsilon)$; $E^0_*(\sigma, \varepsilon) = \alpha E_*(\sigma, \varepsilon)$; $f_0(\varepsilon) = \alpha f(\varepsilon)$, k and b are coefficients, determined experimentally and characterizing the internal friction and cohesion in the soil [10].

For the experimental determination of the above-mentioned mechanical characteristics with short-term dynamic loads, a unit of the quasistatic type has been developed, which has been described earlier in [10, 5]. In its general features, the unit is a vertically standing

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TABLE 1

No.	E_0, kg/cm²	$\frac{k_1}{\nu_1}$	K_0, kg/cm²	$\frac{k_2}{\nu_2}$	σ_1^0, kg/cm²	$\frac{E_{*1}}{E_{*2}}$
1	1000	$\frac{0{,}84\cdot 10^3}{3{,}40}$	150	$\frac{0{,}38\cdot 10^2}{2{,}00}$	15	$\frac{11{,}0\cdot 10^3}{1{,}7\cdot 10^3}$
2	2000	$\frac{2{,}23\cdot 10^3}{3{,}24}$	300	$\frac{0{,}33\cdot 10^2}{2{,}00}$	15	$\frac{7{,}0\cdot 10^3}{1{,}5\cdot 10^3}$
3	500	$\frac{0}{-}$	150	$\frac{0}{-}$	15	$\frac{18{,}5\cdot 10^3}{2{,}0\cdot 10^3}$
4	500	$\frac{0{,}12\cdot 10^3}{3{,}00}$	75	$\frac{0{,}10\cdot 10^3}{3{,}00}$	5	$\frac{4{,}0\cdot 10^3}{1{,}0\cdot 10^3}$

cylinder, at the bottom of which a sample of soil with a height of h_0 is arranged in a rigid ring. The sample is compressed by a metallic piston, to which, through a gasket, the impact of a freely falling weight is transmitted. During the course of a test, the stresses in the sample $\sigma_1(t)$, $\sigma_2(t)$ and the displacement of the piston $u(t)$ are recorded, and the deformation is determined in a quasistatic approximation in accordance with the formula $\varepsilon(t) = u(t)/h_0$. Changes in the rigidity of the gasket, the weight of the falling load, and the height of its fall can create different deformation conditions for the soil sample. An evaluation of the time t_0 required for the establishment of quasistatic deformation conditions, with application to [10, 5], gives a value of $t_0 \approx 0.2t_+$, where t_+ is the time required for a growth of the stresses $\sigma_1(t)$ to a maximal value.

The determination of the functions $f(\varepsilon)$ and $\varphi_*(\varepsilon, \dot{\varepsilon}_*)$ from the results of static tests with $\dot{\varepsilon} \to 0$ presents no difficulties. The limiting dynamic diagrams $\varphi(\varepsilon)$, $\dot{\varepsilon} = \infty$ were determined from a number of soils in [7-9] from the results of the propagation of shock waves in them.

The method for determination of the function $G(\varkappa_1, \varkappa_2, \ldots, \varkappa_k)$ is based on the minimization of some quadratic function

$$D(\varkappa_1, \varkappa_2, \ldots, \varkappa_k) = \sum_{i=1}^{s} \sum_{j=j0}^{n} [\varepsilon_{ij}(\varkappa_1, \varkappa_2, \ldots, \varkappa_k) - \langle\varepsilon_{ij}\rangle]^2 \tag{6}$$

with respect to the unknown parameters $\varkappa_1, \varkappa_2, \ldots, \varkappa_k$.

In (6) $\langle\varepsilon_{ij}\rangle = \frac{1}{m}\sum_{l=1}^{m} \varepsilon_{il}(t_j)$ is the mean value of the deformations with $t = t_j$, from the results of a series of tests, corresponding to determined deformation conditions; m is the number of experiments in a series; n is the number of time intervals in interpretation of the results of the tests in a given series; j_0 is the number of the interval, corresponding to the moment of the establishment of quasistatic deformation; s is the number of repeated loadings of the same sample in a given series; $\varepsilon_{ij}(\varkappa_1, \varkappa_2, \ldots, \varkappa_k)$ is the value of the deformations for a given $t = t_j$, calculated using formula (1) with given functions $\varphi(\varepsilon)$, $\varphi_*(\varepsilon, \dot{\varepsilon}_*)$, $f(\varepsilon)$ and with a load $\langle\sigma_{1ij}\rangle = \frac{1}{m}\sum_{l=1}^{m} \sigma_{1il}(t_j)$, $j = 1, 2, \ldots, n$, given on the basis of experiment.

We give further the results of a determination of the mechanical characteristics for a sandy soil with $\gamma_0 = 1.48$–1.50 g/cm³, $w = 0.05$ and 0.15, and below, for clayey soils, on the basis of test data [8-12]. Analytical dependences for the limiting diagrams of $\varphi(\varepsilon)$, $\dot{\varepsilon} = \infty$, and $f(\varepsilon)$ and $\dot{\varepsilon} = 0$ are obtained in the form

$$\varphi(\varepsilon) = E_0(\varepsilon + k_1\varepsilon^{\nu_1}); \tag{7}$$

$$f(\varepsilon) = K_0(\varepsilon + k_2\varepsilon^{\nu_2}). \tag{8}$$

The corresponding values of the coefficients E_0, k_1, ν_1, K_0, k_2, ν_2 are given in Table 1, where rows 1 and 2 correspond to sandy soils with a volumetric weight of their skeleton $\gamma_0 = 1.50$ g/cm³, and a moisture content by weight $w = 0.05$ (row 1) and $w = 0.12$–0.15 (row 2); row 3 corresponds to loess-type loam with $\gamma_0 = 1.44$–1.47 g/cm³, $w = 0.12$–0.13; row 4 corresponds to loam with $\gamma_0 = 1.60$–1.65 g/cm³, $w = 0.15$. Since, with the method under consideration for determining the limiting dynamic diagrams $\varphi(\varepsilon)$, $\dot{\varepsilon} = \infty$, it was not possible to determine the initial section, for the soils in question it was assumed that $E_0 = \rho_0 a_0^2$, where a_0 is the rate of propagation of shock waves, from the results of measurements [7-9]. The values

TABLE 2

No.	Model of type I			Model of type II		
	$\varkappa$	η	δ	$\varkappa$	η	δ
1	0,40(±0,10)	1,0(∓0,30)	0,10	0,50(±0,30)	0,6(∓0,40)	0,10
2	0,40(±0,05)	1,8(∓0,30)	0,10	0,22(±0,20)	2,8(∓1,40)	0,10
3	0,45(∓0,05)	3,5(∓0,50)	0,10	0,40(±0,10)	2,0(∓0,30)	0,06
4	0,35(±0,05)	2,6(∓0,40)	0,20	0,25(±0,05)	2,8(∓0,60)	0,20

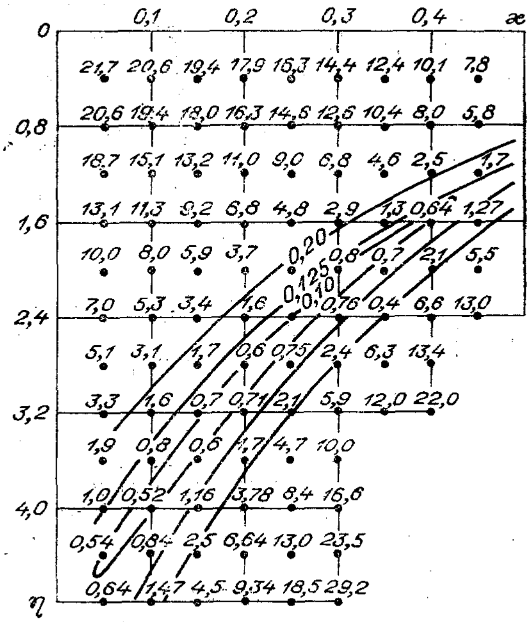

Fig. 1

of the coefficients k_1 and ν_1 in Table 1 were determined, further, from the condition of a maximal approach of (7) to the exponential diagrams obtained in tests [7-9].

The modulus of the deformations with unloading E_* were determined in the form

$$E_* = \begin{cases} E_{*1}, & \sigma_1 > \sigma_1^0, \\ E_{*2}, & \sigma_1 \leqslant \sigma_1^0, \end{cases} \qquad (9)$$

The corresponding values of E_{*1}, E_{*2}, σ_1^0 for the soils investigated are given in Table 1.

The function G, in actual implementation of the method under consideration, is taken in the form

$$G = \eta[\sigma_1 - f(\varepsilon)]^{\varkappa}.$$

Thus, it was assumed that the function G depends on the two parameters $\varkappa_1 = \varkappa$ and $\varkappa_2 = \eta$, with respect to which the function (6) was minimized. In view of the insufficient degree to which the response surface of (6) has been studied, minimization was carried out by the method of residues, in the form of a computer program for a Minsk-32 digital computer.

Two types of models of a viscoplastic medium were examined, determined by the relationship (1) with $E_*(\varepsilon) = E(\varepsilon)$ (model of type I) and with E_* equal to (9) (model of type II).

The search for the values of $(\varkappa, \eta)$ with which the relationship (1) best describes the experiment was carried out in the following order. In the plane $(\varkappa, \eta)$ the values of the function $D(\varkappa, \eta)$ were determined at the lattice points of the grid in accordance with (6). Here it was established that, in all the cases investigated, the sought value of $\varkappa$ is located within the limits $0 < \varkappa < 1$, and of η, within the limits $0 < \eta < 3$-5. Within these limits of the change in the values of $\varkappa$ and η, a rough search was first made, in which a grid with a dimension of 10 × 10 was used. Then, for individual regions of the possible location of minimal values of $D(\varkappa, \eta)$, calculations were made with a small spacing with respect to $\varkappa$ and η. Then interpolation was used to find the level lines corresponding to equal values of the func-

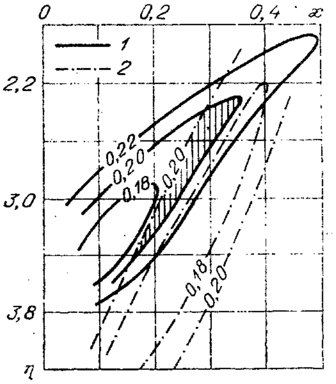

Fig. 2

tion D(ϰ, η). The result was obtained that, in all cases, the level lines for the given deformation conditions have a character close to elliptical.

Specifically, Fig. 1 gives the results of a determination of the level lines for a sandy soil with γ_0 = 1.50 g/cm³, w = 0.15. The numbers at the mesh points correspond to values of D(ϰ, η) · 10³, and the numbers at the level lines to the relative errors $\delta = (D/sn)^{1/2}/\langle\varepsilon\rangle$, where $\langle\varepsilon\rangle = \frac{1}{sn}\sum_{i=1}^{s}\sum_{j=j0}^{n}\varepsilon_i(t_j)$ is the mean value of the deformation for the given process.

Here, on the average, δ characterizes the accuracy with which the calculated curve of $\varepsilon_{ij}(\varkappa, \eta)$ approaches the experimental $\langle\varepsilon_{ij}\rangle$. The criterion of sufficient accuracy of this approximation can be taken as a situation in which the curve of $\varepsilon_{ij}(\varkappa, \eta)$, calculated in accordance with (1) with determined values of (ϰ, η) does not go beyond the confidence intervals for the experimental values of $\langle\varepsilon_{ij}\rangle$ for the given conditions. For this, the value of δ must not exceed the value of the confidence interval of δ_0, averaged over the whole process with respect to the time $\langle\varepsilon_{ij}\rangle$, $j = j_0, j_0 + 1, \ldots, n$. The value of δ_0 for the soil in question lies within the limits δ_0 = 0.05-0.14. Therefore, it can be postulated that a value of δ lying within the limits $0.05 \leq \delta \leq 0.10$ will assure the given accuracy. From the calculations given in Fig. 1 it follows that a given accuracy δ = 0.10 corresponds to some region of values of the pair (ϰ, η). Table 2 (row 2), for the given soil, gives coordinates corresponding to the center of this region, as well as deviations corresponding to its edge points; here analogous results are also given for other soils investigated earlier [10-12].

As follows from Fig. 1, by decreasing the requirement for the accuracy of the experiment it is possible to obtain an arbitrarily large region of values of the pair ϰ, η, satisfying the given accuracy. Lacking data on the accuracy of the experiment, the question of determining characteristics of a soil describing its viscoplastic properties loses all practical meaning, since the region of search for the values of ϰ, η will be indeterminate.

The presence of experimental results corresponding to different deformation conditions for the same soil allows us to narrow considerably the region of sought values of ϰ, η. This is connected with the fact that regions bounded by level lines corresponding to equal values of δ, with different deformation conditions, must have common points; i.e., they must intersect. The lack of common points for such regions is evidence that, with the given accuracy, the model adopted does not give an adequate reflection of the experiment, and, consequently, must be rejected.

Figure 2 gives the results of a determination of the coefficients ϰ, η for loam with γ_0 = 1.60-1.65 g/cm³, w = 0.15, with different deformation conditions (curves 1, 2). The numbers at the level lines correspond to the mean accuracy δ. The hatched region is a region of values of , η, corresponding to δ = 0.2 for both sets of conditions, which lies within the limits of accuracy of the experiments for loam. In Table 2, the regions of values of ϰ, η are determined for models of types I and II. An analysis of these results attests to the fact that, in a range of loads up to 30-50 kg/cm², the models under consideration describe the experiment for sands and loams with a practically identical degree of accuracy. It must be noted that, with a description of process of the damping of explosion waves with short loads, the model of type II is in better agreement with experiment than the model of type I.

From the results obtained (see Table 2) it can be seen that, for sandy and clayey soils, the function G is essentially nonlinear, in distinction from the results of [13].

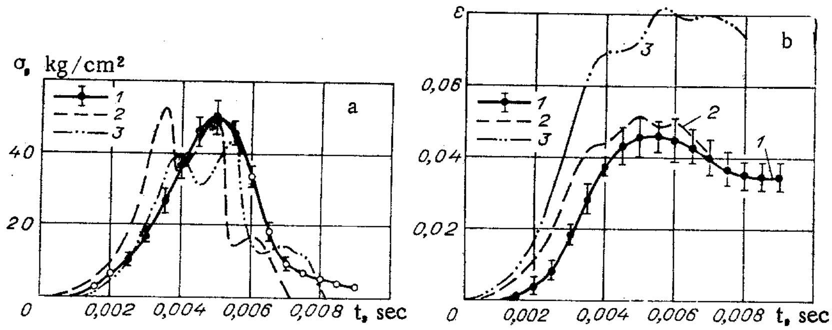

Fig. 3

It is of interest to compare the theoretical curves for $\sigma_1(t)$, $\varepsilon(t)$, calculated with application to the conditions of the experiment, taking account of the data on the characteristics from Tables 1 and 2, with the corresponding experimental curves.

The system of equations describing the work of the experimental unit has the form (the x axis is directed upward)

$$m_0\frac{dv_3}{dt}=\alpha\sigma_3(t),\quad m_2\frac{dv_2}{dt}=\sigma_1(t)-\alpha\sigma_3(t),\ \sigma_3=f_3\left(\frac{u_1-u_3}{l_3}\right),$$
$$\frac{d\sigma_1}{dt}=-E_1(\varepsilon_1)\left[v_1+G(\sigma_1-f(\varepsilon_1))\right],\quad v_2=v_1,\quad v_1=\frac{du_1}{dt},\tag{10}$$
$$v_3=\frac{du_3}{dt},\quad \varepsilon_1=u_1/l_1,\quad E_1(\varepsilon_1)=\begin{cases}E(\varepsilon_1), & \frac{d\sigma_1}{dt}\geqslant 0,\\ E_*, & \frac{d\sigma_1}{dt}<0,\end{cases}$$

where $u_2(t) = u_1(t)$ and $u_3(t)$ are the displacements of the piston (the soil) and the falling weight, respectively; m_0 and m_2 are the mass of the falling weight and the mass of the piston per unit area of the sample; $\sigma_1(t)$ and $\sigma_3(t)$ are the stresses in the sample of soil and the gasket; l_1 and l_3 are the heights of the sample of soil and the gasket; $\alpha = 0.121$ is the ratio of the areas of the gasket and the sample.

The initial conditions for the system of equations (10) are

$$v_3(0)=-V_0(V_0=\sqrt{2gH_0}),\ u_1(0)=u_3(0)=v_1(0)=v_3(0)=0,$$
$$\sigma_1(0)=\sigma_3(0)=0,$$

where H_0 is the height of the fall of the weight.

Figure 3a, b gives the results of calculations of the stresses $\sigma_1(t)$ (curve 2, Fig. 3a) and the deformations $\varepsilon(t)$ (curves 2, Fig. 3b) for sand with $\gamma_0 = 1.50$ g/cm³, $w = 0.05$, with the following data: weight of weight $P_0 = 50$ kg, $H_0 = 13.0$ cm, $l_1 = 3.0$ cm, $l_3 = 0.50$ cm. The diagram of the compression of the gasket $f_3(z)$, $z = (u_1 - u_3)l_3$, was obtained as a result of static experiments, in the form

$$f_3(z)=\begin{cases}125\,z,\ \text{kg/cm}^2, & z\leqslant 0.4,\\ 673+3900\,z^2-3120\,z,\ \text{kg/cm}^2, & z>0.4.\end{cases}$$

Curves 2, Fig. 3a, b correspond to $\varkappa = 0.5$, $\eta = 0.6$ cm/(kg·sec)$^{1/2}$; curves 3 correspond to an elastoplastic model, characterizing the static diagram of the compression $f(\varepsilon_1)$ with loading. Unloading under these conditions is assumed analogously to the viscoplastic model with $\sigma_1 < f(\varepsilon_1)$. Curves 1 represent experimental values of the stresses and deformations, with their own confidence intervals.

As can be seen from Fig. 3a, in both cases the theoretical curves of $\sigma_1(t)$ lie within the limits of the confidence intervals only at individual points. Here the value of the maximal stresses for curve 2 ($\varkappa = 0.5$, $\eta = 0.6$ cm/(kg·sec)$^{1/2}$) practically coincides with the experimental, while, for curve 3 it is 15-20% less.

On the whole it must be noted (with respect to the experimental data) that the curves of $\sigma_1(t)$ are not very sensitive to changes in the parameters of the model $\varkappa$, η. It was precisely in view of this fact that the minimization of the function $D(\varkappa, \eta)$, with determination of the values of $\varkappa$, η, was carried out with respect to the deformations $\varepsilon(t)$.

The theoretical curve 2 in Fig. 3b with t > 0.004 lies within the limits of the confidence intervals for the experimental points, while curve 3, obtained with calculations using an elastoplastic model (without taking account of viscosity), goes considerably beyond the confidence intervals for all moments of time. This is evidence of unsuitability of an elastoplastic model for description of the deformation of a soil with short-term dynamic loads.

Taking into consideration that the rate of deformation does not have any substantial effect on the condition of plasticity, on the basis of the results obtained above on monaxial compression (see Tables 1, 2), it is possible to determine the corresponding characteristics of the soils investigated for volumetric compression, in accordacne with (5).

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